BUCKLING IN THE PRESENCE OF CREEP

(VYPUCHIVANIE PRI POLZUCHESTI)

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The paper investigates the buckling of an idealized bar when a work-hardening theory in the form

$$\dot{p} = Ap^{-\alpha}\sigma^n \qquad \left(p = \varepsilon - \frac{\sigma}{E}\right) \tag{1}$$

is admissible.

We shall compare the results with the solution obtained by linearization of Expression (1) [1,2]. One solution to the problem of longitudinal bending of an idealized bar in accordance with the work-hardening theory has been given by Libov [3], in which another form of Expression (1) was used. Libov carried out his investigation with the aid of numerical methods, and it is therefore not possible to make a comparison with the linearized form of the problem.

Let us consider an idealized bar of *I*-section (see Figure). We shall assume that the bar has an initial deflection and is in compression under the action of a longitudinal force *P*. If we denote the area of a flange by *F* and the depth of the bar by $2h$, we obtain the following expressions for the stresses σ_1 and σ_2 in the flanges (subscript 1 corresponds to the concave flange):

$$\sigma_1 = \sigma_0\left(1 + \frac{y}{h}\right), \qquad \sigma_2 = \sigma_0\left(1 - \frac{y}{h}\right), \qquad \sigma_0 = \frac{P}{2F} \tag{2}$$

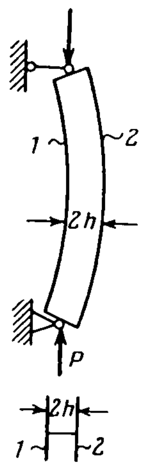
If we accept the plane-sections hypothesis for strains ϵ_1 and ϵ_2 in the flanges, we obtain

$$\varepsilon_1 - \varepsilon_2 = -2h\,(y'' - y_0'') \tag{3}$$

We shall suppose that

$$y = hu\sin\frac{\pi x}{L}, \qquad y_0 = hu_{00}\sin\frac{\pi x}{L} \tag{4}$$

where u is the amplitude of the nondimensional deflection at the centre of the bar.



We shall carry out our investigation satisfying all the equations only at the center of the bar. We then have

$$\frac{2h^2\pi^2}{L^2}\left[(u-u_{00})-\beta u\right]=p_1-p_2 \qquad \left(\beta=\frac{P}{P_э}<1\right) \tag{5}$$

where $P_э$ is the critical load for an elastic bar.

We evaluate the quantity $z = p_1 - p_2$, making use of Expression (1), and find that

$$z=\{A(1+\alpha)\sigma_0{}^n\}^{\frac{1}{1+\alpha}}\left\{\left[\int_0^t (u+1)^n dt\right]^{\frac{1}{1+\alpha}}-\left[\int_0^t (1-u)^n dt\right]^{\frac{1}{1+\alpha}}\right\} \tag{6}$$

For small values of the parameter u we have

$$z^{1+\alpha}\approx A(1+\alpha)\,\sigma_0{}^n\left(\frac{2nu_0}{1+\alpha}\right)^{1+\alpha} t \tag{7}$$

From this we assume

$$\dot{z}z^\alpha\approx A\sigma_0{}^n\left(\frac{2n}{1+\alpha}\right)^{1+\alpha} u^{1+\alpha} \tag{8}$$

For large values of the parameter u we obtain

$$z^{\alpha+1}\approx A(1+\alpha)\sigma_0{}^n 2\int_0^t u^n\,dt+c \qquad \text{or} \qquad \dot{z}z^\alpha\approx A\sigma_0{}^n 2u^n \tag{9}$$

Finally, we have

$$\dot{z}z^\alpha=A\sigma_0{}^n k u^{1+\alpha}(k_1u+1)^{n-1-\alpha} \qquad \left(k=\left(\frac{2n}{1+\alpha}\right)^{1+\alpha},\quad k_1{}^{n-1-\alpha}=\frac{(1+\alpha)^{1+\alpha}}{2^\alpha n^{1+\alpha}}\right) \tag{10}$$

Then, combining (5) and (10), we obtain

$$\frac{(u-u_0)^\alpha\dot{u}}{u^{1+\alpha}(k_1u+1)^{n-1-\alpha}}=A_1 \tag{11}$$

$$\left(A_1=A\sigma_0{}^n k k_2{}^{-1-\alpha},\ k_2=\frac{2h^2\pi^2}{L^2}(1-\beta),\ u_0=\frac{u_{00}}{1-\beta}\right)$$

Thus the problem of finding the relation $u = u(u_0, t)$ is reduced to one of quadrature. The condition $\ddot{u} = 0$ gives the equation

$$k_1(n-\alpha)\,u_*{}^2+(1-k_1u_0)\,u_*-(1+\alpha)\,u_0=0 \tag{12}$$

for finding the critical value u_*.

For $u_0 \ll 1$ we have $u_* = (1+\alpha)u_0$. Consequently, we can neglect k, u in (11) compared to unity, and we then find that

$$A_1 t_* = \int_{u_0}^{u_0(1+\alpha)} \frac{(u-u_0)^\alpha du}{u^{1+\alpha}} \tag{13}$$

It is evident that t_* is independent of u_0. We have

$$t_*(\alpha) = \frac{1}{A_1} \int_1^{1+\alpha} \frac{(x-1)^\alpha\, dx}{x^{\alpha+1}} \tag{14}$$

For convenience, instead of time t, we introduce a strain corresponding to the mean stress, i.e.

$$\dot{p} p^\alpha = A\sigma_0{}^n \tag{15}$$

Then

$$p_* = \left\{\frac{A\sigma_0{}^n}{1+\alpha} t_*\right\}^{\frac{1}{1+\alpha}} \tag{16}$$

or

$$p_* = \frac{k_2}{2n}(\alpha+1)\Psi(\alpha), \qquad \Psi(\alpha) = \left[(1+\alpha)\int_1^{1+\alpha} \frac{(x-1)^\alpha\, dx}{x^{1+\alpha}}\right]^{\frac{1}{1+\alpha}} \tag{17}$$

Since for the bar in question $\varepsilon_0 = h^2 a^2/L^2$ and $\epsilon = \sigma_0/E$, we can write

$$p_{1*} = \frac{np_*}{\varepsilon_0 - \varepsilon} = (1+\alpha)\Psi(\alpha) \tag{18}$$

Expression (18) is analogous to Formula (16) in [2], in which p also is a function only of α. For the case when $\alpha = 1$ we have

$$p_1 = 1.36, \qquad p_{1*} = 1.32 \tag{19}$$

Expressions (18) and (19) show that the criterion that $\ddot{u} = 0$, which was introduced in the linearized problem, leads to the same results in the nonlinear problem in the case of small u_0. We can compare the values of t_* and t_∞ derived from the condition that $u \to \infty$.

From (11) we obtain

$$t_\infty = \frac{1}{A_1}\int_{u_0}^{\infty} \frac{(u-u_0)^\alpha\, du}{(1+k_1 u)^{n-1-\alpha} u^{1+\alpha}} \tag{20}$$

Evidently, $t_\infty \to \infty$ as $u_0 \to 0$ and always $t_\infty > t_*$. Consider now the parameter p. We have

$$\lambda = \left(\frac{p_{1\infty}}{p_{1*}}\right)^{1+\alpha} = \left(\int_0^\infty \frac{(u-u_0)^\alpha\, du}{u^{\alpha+1}(k_1 u+1)^{n-1-\alpha}}\right)\left(\int_0^{u_*} \frac{(u-u_0)^\alpha\, du}{u^{1+\alpha}(k_1 u+1)^{n-1-\alpha}}\right)^{-1} \tag{21}$$

where u_* is given by Equation (12).

For values of $n = 3$, $\alpha = 1$ from (21) for u_0 equal to 0.001, 0.01, 0.1, 1 and 10 we have the values of 40, 25, 14, 7 and 1.5, respectively. For the same values of u_0 the parameter (p_1/p_{1*}) has the values 1, 1.03, 1.35, 10. From these numerical results it follows that the criterion $\ddot{u} = 0$ for nonlinear and linearized problems gives practically coincident results for $u_0 < 1$. Obviously, for a real bar with an initial deflection u_0 the condition $u \to \infty$ gives an upper bound and the condition $u = 0$ gives a lower bound.

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